APPROXIMATE TIDE-CONSTANTS FOR TABLE BAY AND ALGOA BAY.

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At the suggestion of H.M. Astronomer, D. Gill, Esq., self-registering tide-gauges of the most approved pattern were erected in 1880 at Cape Town and Port Elizabeth by the respective Harbour Boards: and the tracings of several years' tides are now available for reduction. It is to be hoped that arrangements will soon be made for a complete discussion of these curves and for an accurate prediction of future tides. Meanwhile I have thought it would be of considerable interest to obtain some preliminary results with regard to the more important of the tidal constituents, and to see in the case of Table Bay whether any correction is indicated to the data published in the *Shipping Gazette*, 1853, and used for the formation of the tide-tables to the present day. These data were derived from 4,099 observations of the time of high water at the Naval Yard, Simon's Town, during the years 1840-1848. From previous observations in 1834 and 1835 with two temporary gauges, one at Simon's Bay and the other at the South Jetty, Table Bay (near the Imhoff Battery), the conclusion was arrived at that there was no sensible difference between the time of high water in Simon's Bay and in Table Bay: and the results of the Simon's Bay observations were adopted without alteration. No information, however, is given in the *Shipping Gazette* about the heights of the different high waters. Tide-gauge diagrams have, I believe, been regularly filed for many years previous to 1880, but no attempt (so far as I know) has ever been made to discuss them.

The height of the tide at any place may be expressed as the sum of a number of simple harmonic functions of the time, the periods of the functions being the same as the periods of certain terms in the theory of the Sun and Moon. The analytical expression for such a function is

$$\mathrm{A} \cos nt + \mathrm{B} \sin nt, \text{ or } \mathrm{R} \cos (nt - \varepsilon),$$

where t denotes the time elapsed from some starting-point, n is the 'speed' of the tide (so that the function repeats itself in a time $\left.\frac{2\pi}{n}\right)$, R is the amplitude and ϵ the epoch. Each such component is called a tide. The actual height of the tide therefore at any time t is equated to the expression

$$A_0 + R_1 \cos (n_1 t - \epsilon_1) + R_2 \cos (n_2 t - \epsilon_2) + \&c.$$

where A_0 is the height of mean sea-level above some assumed datum-line ; and the solution of a large number of equations formed in this way determines the most probable values of the constants involved.

Prof. G. H. Darwin has shown that it is possible to deduce very approximate values of six of the most important tides from observations taken every hour for a month. These are the lunar, solar and luni-solar semi-diurnal tides ; and the luni-solar, lunar and solar diurnal tides : they will be referred to by the letters M_2, S_2, K_2, K_1, O and P respectively. Of these the two first are usually much the largest and most important, and they are the only ones that can be taken into account in forming a *mean* tide-table to apply approximately throughout the year. But it should be understood that such a table must necessarily be rough : the diurnal tides are neglected and will have the effect of making the predicted times of high water differ from the observed times by about the same amount for the morning and evening tides but in different directions, and there will be inequalities depending on the moon's declination and distance from the earth. If it be thought worth while to allow for these inequalities, the results of the present investigation will allow of its being done : but as a month's observations are not sufficient to determine the elliptic tides and as the time and height of high water are to some extent dependent on the meteorological conditions obtaining at the time, it is perhaps scarcely worth while to incur the extra labour of the computations. For Table Bay I have worked up two series, one in 1885, November, and one in 1887, May : the results from the two series showed a most satisfactory agreement. For Algoa Bay I have taken one series in 1886, April. I have assumed that the height of the tide is affected to the extent of one foot by a change of one inch in the barometer reading, and corrections have been applied to reduce the readings to a mean height of the barometer of 30 inches.

Putting H for the semi-range of a tide, and K for the constant angle of retardation or 'lag,' I find

TABLE I.

	TABLE BAY.		ALGOA BAY.	
'Age' of tide	$= 26^{\mathrm{h}}{\cdot}4$		$= 32^{\mathrm{h}}{\cdot}0$	
Mean height above datum-line	$= 2{\cdot}37$ ft		$= 2{\cdot}34$ ft	
	$M_2 \begin{cases} H_m = 1{\cdot}60 \text{ ft} \\ K_m = 67^\circ \end{cases}$	$K_1 \begin{cases} H' = 0{\cdot}19 \text{ ft} \\ K' = 124^\circ \end{cases}$	$M_2 \begin{cases} H_m = 1{\cdot}76 \text{ ft} \\ K_m = 97^\circ \end{cases}$	$K_1 \begin{cases} H' = 0{\cdot}16 \text{ ft} \\ K' = 146^\circ \end{cases}$
	$S_2 \begin{cases} H_s = 0{\cdot}68 \text{ ft} \\ K_s = 93^\circ \end{cases}$	$O \begin{cases} H_o = 0{\cdot}06 \text{ ft} \\ K_o = 250^\circ \end{cases}$	$S_2 \begin{cases} H_s = 0{\cdot}83 \text{ ft} \\ K_s = 128^\circ \end{cases}$	$O \begin{cases} H_o = 0{\cdot}05 \text{ ft} \\ K_o = 280^\circ \end{cases}$
	$K_2 \begin{cases} H'' = 0{\cdot}18 \text{ ft} \\ K'' = 93^\circ \end{cases}$	$P \begin{cases} H_p = 0{\cdot}06 \text{ ft} \\ K_p = 124^\circ \end{cases}$	$K_2 \begin{cases} H'' = 0{\cdot}23 \text{ ft} \\ K'' = 128^\circ \end{cases}$	$P \begin{cases} H_p = 0{\cdot}05 \text{ ft} \\ K_p = 146^\circ \end{cases}$
'Mean' establishment	$= 2^{\mathrm{h}}\ 19^{\mathrm{m}}$		$= 3^{\mathrm{h}}\ 21^{\mathrm{m}}$	

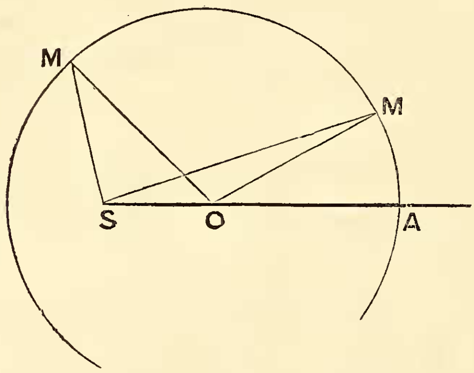
The diurnal tides are therefore small at both places, but they are quite sensible.

The 'mean' establishment hitherto used for Table Bay is $2^{\mathrm{h}}\ 43^{\mathrm{m}}$, so that the considerable correction of 24^{m} must be applied to the average of the predicted times of high water. This correction to the old value may be due partly to the different position of the gauge in the Bay, but more probably to an error in the statement that the times of high water at the Naval Yard, Simon's Bay, and at the South Jetty, Cape Town, are coincident: for unless special precautions are taken in the erection of a tide-gauge to annul the effects of wave-disturbance it is no easy matter to determine the time of high water from the diagram to a good many minutes.

The 'vulgar' establishment, or time of high water at full and change of the Moon, is $2^{\mathrm{h}}\ 33^{\mathrm{m}}$. The 'vulgar' establishment given for Algoa Bay on the charts is 4^{h}, but Capt. Skead, R.N., has pointed out that the observations on which this result depends were taken at a station some way up the river and that the value for the Bay should be considerably less: my results make it $3^{\mathrm{h}}\ 39^{\mathrm{m}}$.

A 'mean' tide-table, sufficient for purposes of approximately predicting the time of high water, can easily be formed by a graphical process from the above results for the M. and S. tides.

The local mean time of the Moon's transit is to be converted into angle at the rate of 30° to the hour and from it the angle ($K_s - \frac{30}{29} K_m$) subtracted : call the result x. Then on any convenient scale take a straight line SO equal to H_s and produce it to any point A. With O as centre and H_m as radius describe a circle : and from the point O draw a straight line OM making with AO an angle equal to x and cutting the circle in M.



Then SM is the height of the tide, and the angle OMS converted to time at the rate of 29° to the hour is the correction to be applied to the mean establishment to find the interval by which the time of high water follows the moon's transit over the meridian.

The correction is subtractive if x be less than 180°, additive if greater.

Table II is such a table ; it gives for Table Bay and Algoa Bay the time of high water and the height of the tide above the datum-line corresponding to each half hour of the Moon's time of passing the meridian. The hours of the Moon's transit are reckoned from 0^h to 24^h, commencing at noon : and it is to be understood that when the time of transit is greater than 12^h the corresponding time of high water is to be increased by 12^h.

TABLE II.

Local Time of Moon's Transit.	Table Bay.		Algoa Bay.	
	Time of High Water.	Height of Tide.	Time of High Water.	Height of Tide.
h m h m	h m	ft	h m	ft
0 0 or 12 0	2 33	4·61	3 39	4·87
0 30 or 12 30	2 54	4·65	3 59	4·92
1 0 or 13 0	3 15	4·65	4 20	4·93
1 30 or 13 30	3 36	4·61	4 40	4·91
2 0 or 14 0	3 57	4·55	5 0	4·84
2 30 or 14 30	4 19	4·46	5 21	4·71
3 0 or 15 0	4 41	4·34	5 42	4·61
3 30 or 15 30	5 4	4·19	6 4	4·45
4 0 or 16 0	5 29	4·03	6 28	4·26
4 30 or 16 30	5 57	3·86	6 54	4·05
5 0 or 17 0	6 28	3·71	7 23	3·84
5 30 or 17 30	7 5	3·51	7 57	3·63
6 0 or 18 0	7 48	3·38	8 38	3·44
6 30 or 18 30	8 37	3·30	9 28	3·31
7 0 or 19 0	9 30	3·30	10 25	3·07
7 30 or 19 30	10 20	3·37	11 20	3·34
8 0 or 20 0	11 3	3·50	12 8	3·49
8 30 or 20 30	11 40	3·70	12 46	3·68
9 0 or 21 0	12 11	3·84	13 19	3·91
9 30 or 21 30	12 39	4·02	13 47	4·11
10 0 or 22 0	13 4	4·18	14 13	4·31
10 30 or 22 30	13 28	4·33	14 36	4·50
11 0 or 23 0	13 50	4·45	14 58	4·65
11 30 or 23 30	14 12	4·55	15 19	4·78

If we desire to take account of the parallactic and declinational corrections we may proceed as follows :—

For SO (in above fig.) take the value $1{\cdot}086\ p_{,} \cos^2 \delta_{,}\ H_s$ where $p_{,}$ is the cube of the ratio of the Sun's parallax at the time under consideration to the mean parallax, and $\delta_{,}$ is the Sun's declination. Then from Tables III and IV take the small corrections depending on the Moon's parallax and declination, remembering that these corrections are to be taken out for the Moon's position at a time anterior to the high water by the 'age.'

TABLE III.

Argument—Minutes of Moon's Parallax in excess or defect of 57′ 2″.

′	
	ft
1	$\delta_{,}\ R_m = \pm\ 0{\cdot}08$
2	$0{\cdot}17$
3	$0{\cdot}25$
4	$0{\cdot}34$
5	$0{\cdot}42$

TABLE IV.

Argument—Moon's Declination.

☾'s Dec. N. or S.	$\delta_2\ R_m$.	$\delta_2\ K_m$.	$\delta_2 \iota$.
°	ft	°	h
0	$+0{\cdot}12$	$+2{\cdot}0$	$+0{\cdot}07$
5	$0{\cdot}11$	$1{\cdot}8$	$\cdot 06$
10	$0{\cdot}08$	$1{\cdot}3$	$\cdot 05$
15	$+0{\cdot}02$	$+0{\cdot}4$	$+\ \cdot 01$
20	$-0{\cdot}05$	$-0{\cdot}8$	$-\ \cdot 03$

The height and time of high water can be found from the figure (as in the general case) if we substitute

$$H_m + \delta_1 R_m + \delta_2 R_m \text{ for OM}$$
$$x + \delta_2 K_m \text{ for the angle } x$$

and add the time $\delta_2 \iota$ to the mean establishment. We can also easily find an approximate value of the largest diurnal tide :—

Let t be the time of H.W. and h the height of any afternoon tide computed from the mean tide-table. Find ζ' ($=$ K′ $-$ Sun's long. $+$ 90°) and subtract it from t converted into angle at the rate of 15° to the hour : call the remainder λ : then $\dfrac{H' \operatorname{Sin} \lambda}{h}$ is the correction to t in decimals of an hour.

and $H' \cos \lambda$ is the correction to h.

The corrections to the morning tide may be taken as the same in amount but of opposite sign.